Sec 7.3 Solving Exponential Equations

1. Review: The Laws of Exponents

<table>
<thead>
<tr>
<th>Exponential Law</th>
<th>Rule</th>
<th>Example</th>
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<tbody>
<tr>
<td>1. Product Law</td>
<td>( x^m \times x^n = )</td>
<td>( 2x^2 \cdot 3x^3 = )</td>
</tr>
<tr>
<td>2. Quotient Law</td>
<td>( x^m \div x^n = )</td>
<td>( 12x^{12} \div 3x^3 = )</td>
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<tr>
<td>3. Power of a Power</td>
<td>( (x^m)^n = )</td>
<td>( (2x^3)^4 = )</td>
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<tr>
<td>4. Power of a Quotient</td>
<td>( \frac{x^m}{y^n} = )</td>
<td>( \left(\frac{2x}{5y}\right)^3 = )</td>
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<tr>
<td>5. Negative Exponent</td>
<td>( x^{-m} = )</td>
<td>( 13^{-2} = )</td>
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<td>( \left(-\frac{3}{4}\right)^{-3} = )</td>
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<tr>
<td>6. Zero Exponent</td>
<td>( x^0 = )</td>
<td>( x^0 + y^0 - 5 = )</td>
</tr>
<tr>
<td>7. Rational Exponent</td>
<td>( \frac{m}{x^n} = )</td>
<td>( \frac{2}{8^3} = )</td>
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Example 1 – Simplify \( \frac{(2x^2y^3)^2(x^5y)^3}{(xy^2)^4(3x^5y^3)^2} \).

2. Changing Base of Powers

Examples 2 – Write each expression as a power with base 2.

a) \( 4^3 \)  

b) \( \frac{1}{8} \)  

c) \( 8^2 \left(\sqrt{16}\right)^3 \)
3. **Solving Exponential Equations by Using a Common Base**

Use the property that if $c^x = c^y$, then $x = y$ for $a$, $b$, $c$.

**Examples 3** – Solve for $x$.

a) $2^{4x} = 4^{x+3}$

b) $9^{4x} = 27^{x-1}$

c) $5^{2x+3} = 1$

d) $25^{13x} = \frac{1}{125^x}$

e) $\frac{1}{8^3}(4)^x = \frac{1}{16^x}$

f) $\frac{16^{2x}}{8^{x+2}} = 32^x - 4$
4. Growth and Decay Problems

Examples 4

a) A bacterium is quadrupling every seven days. How many times as great will the number of bacteria be in 3 weeks as the number now?

b) In 2 min, a sample of radium-221 decays to 6.25% of its original amount. What is its half-life?

c) After 30 hrs, a sample of plutonium-243 decays to \( \frac{1}{64} \) of its original amount. What is its half-life?

d) At initial count, a bacteria culture contained 1250 bacteria. Another count, 1.5 hrs later, revealed about 80,000 bacteria. What is the doubling period for this bacterium?
5. **Compound Interest**

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