Sec 8.3 – Laws of Logarithms

Since logarithms are exponents, the laws of logarithms are related to the laws of powers.

1. **Helpful Logarithmic Rules**
   a) Use the identity to \( c^x = c^y \) to derive a helpful logarithmic rule.

   b) Use the identity to \( \log_c x = \log_c x \) to derive some helpful logarithmic rule.

2. **Laws of Logarithms**

<table>
<thead>
<tr>
<th>Name</th>
<th>Law</th>
<th>Example – Evaluate.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Multiplication</strong></td>
<td>( \log_c(xy) = )</td>
<td>( \log_6 2 + \log_6 3 )</td>
</tr>
<tr>
<td><strong>Division</strong></td>
<td>( \log_c \left(\frac{x}{y}\right) = )</td>
<td>( \log_2 24 - \log_2 3 )</td>
</tr>
<tr>
<td><strong>Power</strong></td>
<td>( \log_c(x^n) = ) ( \log_c(x^n) = )</td>
<td>( \log_3 25 + \log_3 625 )</td>
</tr>
</tbody>
</table>

3. **Use the Laws of Logarithms to Expand Expressions**

   **Your Turn (pg 395)** – Write each expression in terms of individual logarithms of \( x, y, \) and \( z. \)

   a) \( \log_6 \frac{x}{y} \)                       b) \( \log_5 \sqrt{xy} \)

   a) \( \log_3 \frac{9}{\sqrt{x^2}} \)           b) \( \log_7 \frac{x^3 y}{\sqrt{z}} \)
4. **Use the Laws of Logarithms to Evaluate Expressions**

**Examples 1** – Use the laws of logarithms to simplify and evaluate each expression.

a) \( \log_4 32 + \log_4 2 \)  

b) \( \log_5 500 – \log_5 4 \)  

c) \( \log 4 + \log 5 – \log 2 \)

* Note:

**Your Turn (pg 396)** – Use the laws of logarithms to simplify and evaluate each expression.

a) \( \log_3 9\sqrt{3} \)  

b) \( \log_5 1000 – \log_5 4 – \log_5 2 \)  

c) \( 2\log_3 6 – \frac{1}{2} \log_3 64 + \log_3 2 \)

5. **Use the Laws of Logarithms to Simplify Expressions**

**Examples 2** – Write each expression as a single logarithm in simplest form.

a) \( \log x + \log y – \log z \)  

b) \( \log m – \log n – \log p \)

\[ c) \ 2\log a + 5\log b \]  
\[ d) \ 3\log a – \frac{1}{2} \log b + \frac{5}{4} \log c \]

* Note:
Your Turn (pg 397) – Write each expression as a single logarithms in simplest form. State the restrictions on the variable.

a) \(4\log_3 x - \frac{1}{2}(\log_3 x + 5\log_3 x)\)  
b) \(\log_2(x^2 - 9) - \log_2(x^2 - x - 6)\)

Examples 3 – If \(x = \log_2 5\) and \(y = \log_2 3\), write the following in terms of \(x\) and \(y\).

a) \(\log_2 625\)  
b) \(\log_2 15\)  
c) \(\log_2 300\)

Example 4 – If \(\log_x 5 = a\) and \(\log_x 9 = b\), determine an expression for \(\log_x \left(\frac{25}{3}\right)\) in terms of \(a\) and \(b\).

6. Solving Problems Involving a Logarithmic Scale

The decibel scale (measurement for the intensity of sound) and the pH scale (measurement for the acidity or alkalinity of a solution) both use a logarithmic scale.
Example 5 – The human ear is sensitive to a large range of sound intensities. Scientists have found that the sensation of loudness can be described using a logarithmic scale. The intensity level, $\beta$, in decibels of a sound is defined as $\beta = 10 \log \frac{I}{I_0}$, where $I$, is the intensity of the sound, in watts per square metre (W/m$^2$), and $I_0$ is $10^{-12}$ W/m$^2$, corresponding to the faintest sound that can be heard by a person of normal hearing.

a) Determine the sound level, in decibels, that is 20 times more intense than $I_0$.  
b) The level of sound in a quiet bedroom at night might be 30 dB, while normal conversation has a sound level of about 60 dB. How many times more intense is normal conversation than the quiet room?

Your Turn (pg 399) – The pH scale is used to measure the acidity or alkalinity of a solution. The pH of a solution is defined as $pH = -\log[H^+]$, where $[H^+]$ is the hydrogen ion concentration in moles per litre (mol/L). A neutral solution, such as pure water, has a pH of 7. Solutions with a pH of less than 7 are acidic and solutions with a pH of greater than 7 are basic or alkaline. The closer the pH is to 0, the more acidic the solution is.

A common ingredient in cola drinks is phosphoric acid, the same ingredient in many rust removers. A cola has a pH of 2.5. Milk as a pH of 6.6. How many times as acidic as milk is a cola drink?